Progress Report for Department of Energy Grant DE-FG02-03ER46080 "Heat Transport by Turbulent Rayleigh-Bénard Convection" Sept.15 2003 to March 13 2008

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I. INTRODUCTION

Turbulent Rayleigh-Bénard convection (RBC) in a fluid heated from below is an important process in many respects. It occurs naturally in Earth's atmosphere and oceans and thus has short-term as well as long-term impact on climate and weather. It takes place in Earth's mantle where it affects the motion of continental plates and vulcanism. It drives Earth's dynamo and determines the magnetic field by convection in the liquid outer core. It is the important heat transport mechanism in the outer layer of the Sun and thereby influences the climate on Earth. It plays a significant role in many industrial processes, where its enhancement or inhibition may have significant economic consequences. It is a common phenomenon in everyday life which provokes fascination for the non-specialist. Last, but not least, it is one of the challenging and largely unsolved problems in nonlinear physics. With support from the Department of Energy we have developed a research program dedicated to the quantitative laboratory study of turbulent convection.

Our work was received extremely well by the scientific community, as evidenced by referees' comments on our publications. Below at the relevant places we quote some of these comments in *italics*.

In what follows we shall refer to those of our papers based on work supported by this grant as (1), (2), etc. These papers are listed in Sect. X. Papers by others are referenced as [1], [2], etc. They are listed in Sect. XIII

II. HEAT TRANSPORT BY TURBULENT RAYLEIGH-BÉNARD CONVECTION

An important aspect of turbulent RBC is the global heat transport that is usually expressed in terms of the Nusselt number $\mathcal{N} = QL/\lambda\Delta T$ (Q is the heat-current density, L the sample height, ΔT the applied temperature difference, and λ the thermal conductivity of the fluid in the absence of convection). A central prediction of various theoretical models [1] is a relationship between $\mathcal{N}$, the Rayleigh number R, and the Prandtl number σ. We have now completed our precision measurements of $\mathcal{N}$ as a function of R using water with a Prandtl number $\sigma = 4.4$ in cylindrical samples of various aspect ratios.

A. Effect of finite top and bottom plate conductivity on $\mathcal{N}$

Three apparatus, known as the large, medium, and small apparatus, are described in (4). In that publication we also describe our results illustrating the influence of the finite conductivity of the top and bottom plates on the heat transport in the fluid. For the large and medium sample the results with Aluminum plates fall below those obtained with Copper plates, thus confirming qualitatively the prediction by Verzicco [2] that plates of finite conductivity diminish the heat transport in the fluid. All previous investigations in this field had been unaware of this effect. The Nusselt number $\mathcal{N}_\infty$ for plates with infinite conductivity was estimated by fitting simultaneously Aluminum- and Copper-plate data sets to an appropriate functional form multiplied by a correction factor $f(X) = 1 - exp[-(aX)^b]$ that depends on the ratio X of the thermal resistance of the fluid to that of the plates as suggested by Verzicco.

B. Results for $\Gamma < 1$

Results for $\mathcal{N}_\infty$ as a function of R for cylindrical samples of water (Prandtl number $\sigma = 4.4$) and aspect ratios $\Gamma = D/L \leq 1$ (D is the diameter and L the height) were published in (3). For each aspect ratio $D/L = 0.28, 0.43, 0.67,$ and 0.98 the data cover a range of a little over a decade of R. The maximum $R \simeq 10^{12}$ and Nusselt number $\mathcal{N}_\infty \simeq 600$

were reached for $\Gamma = 0.43$. The results for $\mathcal{N}_\infty$ and $\Gamma \geq 0.43$ are nearly independent of Γ. For $R \lesssim 10^{11}$, the effective exponent γ_{eff} of $\mathcal{N}_\infty = N_0 R^{\gamma_{eff}}$ is about 0.32, larger than those of the Grossmann-Lohse model with its current parameters by about 0.01. For the largest Rayleigh numbers covered for $\Gamma = 0.98$, 0.67, and 0.43, γ_{eff} saturates at the asymptotic value $\gamma = 1/3$ of the Grossmann-Lohse model. This saturation at *finite* R can not be reproduced by the theory, and shows that our theoretical understanding of turbulent convection is still incomplete. Our data do not reveal any crossover to a Kraichnan regime [3] with $\gamma_{eff} > 1/3$ as had been found by others [4]. The reason for this difference between different experiments is an active topic of discussion in the community.

We cite here the comments of a kind referee about paper (3):
The impressively high precision of the data and the small deviations from theory show that these measurements now cross the border into a range, in which more details of the flow become visible also in the global transport properties. This opens the possibility to try to include such details into theory.

C. Results for $\Gamma \geq 1$

Results for $\mathcal{N}_\infty(R)$ and aspect rations $\Gamma = D/L \geq 1$ were published in (5). We used samples with diameters $D = 49.7, 24.8$, and 9.2 cm, all with an aspect ratio $\Gamma \equiv D/L$ (L is the sample height) close to one. In addition, we obtained data for $D = 49.7$ and $\Gamma = 1.5, 2, 3$, and 6. For each sample the data cover a range of a little over a decade of R. Together they span the range $10^7 \lesssim R \lesssim 10^{11}$, with $\mathcal{N}$ ranging from 20 to 280. The effective exponent γ_{eff} of $\mathcal{N}_\infty = N_0 R^{\gamma_{eff}}$ ranges from 0.28 near $R = 10^8$ to 0.333 near $R \simeq 7 \times 10^{10}$. The results are compared to the predictions of a model by Grossmann and Lohse (GL). For $R \lesssim 10^{10}$ they are consistent with the theory. For the largest Rayleigh numbers covered γ_{eff} saturates at the large-R *asymptotic* value $\gamma = 1/3$ of the Grossmann-Lohse model. The data for $\Gamma > 1$ are only a few percent smaller than the $\Gamma = 1$ results.

We cite here the comments of a kind referee about paper (5):
The data are a treasure for the community ... and will serve as benchmark for years, if not decades. The authors did a great service to the community. The achieved precision of the data is remarkable.

D. Effect of departures from the Oberbeck-Boussinesq approximation

In a collaboration with S. Grossmann of Marburg, Germany and D. Lohse of Twente, Netherlands (two theorists) we investigated the effect of temperature dependent fluid properties (non-Oberbeck-Boussinesq or NOB effects) on $\mathcal{N}(R)$ both experimentally and theoretically. This work has been published in (8). In the experiment, the heat current, the temperature difference, and the temperature at the horizontal mid-plane were measured. Three samples of different heights L, all filled with water and all with aspect ratio Γ close to 1, were used. For each L, about 1.5 decades in Ra were covered, together spanning the range $10^8 \leq R \leq 10^{11}$. For the largest temperature difference between the bottom and top plates of $\Delta = 40$K the kinematic viscosity and the thermal expansion coefficient, due to their temperature dependence, varied by more than a factor of two. The Oberbeck-Boussinesq (OB) approximation of temperature independent material parameters thus was no longer valid. The ratio χ of the temperature drops in the bottom and top thermal boundary layer became as small as $\chi = 0.83$, as compared to the ratio 1 in the OB case. Nevertheless, the Nusselt number $\mathcal{N}$ was found to be only slightly smaller (at most 1.4%) than in the next larger sample with the same Rayleigh number, where the material constants were still nearly height-independent. Thus $\mathcal{N}$ is rather insensitive against even significant deviations from OB conditions. The explanation is found in the fact that the reduced thermal input into the bulk from the thinner bottom thermal BL with a smaller temperature drop as compared to the OB case is nearly compensated through the enhanced thermal input from the thicker top BL with a larger temperature drop, so that the net NOB effects on $\mathcal{N}$ remains minor. This study of NOB effects would not have been possible with the typical precision of a percent or two of previous work by others, and was enabled by our resolution of about 0.1 percent.

E. Convection in compressed gases

Very recently we built a small apparatus (sample diameter and height about 7 cm) for the measurement of $\mathcal{N}(R)$ in compressed gases where $\sigma \simeq 1$. This is a difficult problem because the conductivity of the gases is generally small and that of the side wall (which has to be able to sustain the gas pressure of up to 60 bars) is large. Recently we realized that a simple correction for the side wall based on an empty cell is not adequate because the temperature field in the wall varies non-linearly when the cell is filled with convecting fluid. [5] Thus, we do not expect to get data of high accuracy from this investigation, but we do have results of high precision. This permits us to address the NOB problem for this case of smaller Prandtl number $\sigma \simeq 1$ and different departures of fluid properties from a temperature-independent value. It turns out that NOB effects in gases cause $\mathcal{N}(R)$ to increase slightly above the OB value, whereas in liquids like water we had found a slight decrease. It will be a challenge to the theory to explain this observation. This work is as yet unpublished, but will be presented at the March meeting of the APS.

III. PLUMES AND THE LARGE-SCALE CIRCULATION

The emission of hot volumes of fluid, known as "plumes", from the top and bottom boundary layers of the system plays an important role in the heat transport process. These plumes rise under the influence of gravity, and are carried by and in turn drive a large-scale circulation (LSC) of fluid in the entire system. We used the time correlation of shadowgraph images to determine the angle Θ of the horizontal component of the plume velocity just above (below) the center of the bottom (top) plate of a cylindrical Rayleigh-Bénard cell of aspect ratio $\Gamma = 1$ in the Rayleigh-number range $7 \times 10^7 \leq R \leq 3 \times 10^9$ using methanol with Prandtl number $\sigma = 6$. We expect that Θ represents the direction of the LSC. We found that it oscillates time-periodically. The angles near the top and bottom plate have the same frequency but are anti-correlated. The results show that the LSC is a more complicated dynamical system than had been anticipated. Nonetheless, the plume frequency together with the cell dimensions yield a Reynolds number R_e that agrees with measurements by others [6] using completely different techniques. This work was published in Physical Review Letters (2). Additional measurements at other Γ and σ have been carried out since this publication. They yield information about the scaling of R_e with R and σ predicted by theory [1] but not available from experiment heretofore.

We cite the comments of a kind referee about (2):
This careful experimental work is very interesting and significantly advances the knowledge in the field.

IV. REYNOLDS NUMBERS OF THE LARGE-SCALE CIRCULATION

Very recently we installed eight sensitive thermistors in the side walls of the large and medium apparatus, equally spaced in the azimuthal direction at the horizontal mid-plane. These temperature sensors detect the warm up-welling and cold down-flowing fluid without providing any intrusion into the fluid itself that might disturb the large-scale flow. Using correlation-function techniques we were able to determine Reynolds numbers R_e for the flow at values of R an order of magnitude larger than had been done before. This determination of R_e is based on the motion of the plumes, which is generally considered to occur synchronously with that of the LSC. Our data cover the Rayleigh-number range $2 \times 10^8 \lesssim R \lesssim 10^{11}$ and Prandtl-number range $3.3 \lesssim \sigma \lesssim 29$ for cylindrical samples of aspect ratio $\Gamma = 1$. For $R \lesssim R_c \simeq 3 \times 10^9$ we found $R_e \sim R^{\beta_{eff}}$ with $\beta_{eff} \simeq 0.46 < 1/2$. Here both the σ- and R-dependences are quantitatively consistent with the Grossmann-Lohse (GL) prediction. For $R > R_c$ we found $R_e = 0.106\ \sigma^{-3/4} R^{1/2}$, which differs from the GL prediction. The relatively sharp transition at R_c to the large-R regime suggests a qualitative and sudden change that renders the GL prediction inapplicable. However, the question now arises whether the Plume Reynolds number continues to coincide with the LSC velocity Reynolds number..

V. AZIMUTHAL ORIENTATION OF THE LSC

Fitting the eight azimuthally placed temperature readings to a sine function, we were able to determine the orientation θ_0 of the circulation of the large-scale flow. We find that $\theta_0(t)$ shows erratic time dependence that can be described well as azimuthal diffusion.

VI. EFFECT OF EARTH'S CORIOLIS FORCE ON THE LSC

On average the LSC circulation plane rotates in the clockwise direction when viewed from above through about one revolution every three days. The probability distribution of the azimuthal orientation θ_0 has a peak in a location close to West in the laboratory frame. We explored whether this is associated with the Coriolis force due to Earth's rotation. We developed a theoretical model for the interaction between the LSC and the Coriolis force and compared it with the experimental data. Using a Fokker-Planck approach, the model reproduced the probability distribution quantitatively and explained the rotation at the rate of about 0.3 rotations per day. This work was published as (10).

VII. SUDDEN ORIENTATION CHANGES OF THE LSC

We studied the nature of sudden flow reorientations that take place occasionally and that had been observed previously by others [7, 8]. This work was published as paper (6). We used our medium and large cylindrical cells of aspect ratio 1 with the side-wall thermometers to determine the LSC azimuthal orientation $\theta_0(t)$. $\theta_0(t)$ undergoes irregular reorientations. It contains reorientation events by rotation through angles $\Delta\theta$ with a monotonically decreasing probability distribution $p(\Delta\theta)$, and by cessations (where the LSC stops temporarily) with a uniform $p(\Delta\theta)$. Reorientations have Poissonian statistics in time. These results indicate that successive events are independent of each other. The amplitude δ of the LSC and azimuthal rotation rate $|\dot{\theta}_0|$ for reorientations are related by a power law, with smaller δ at larger $|\dot{\theta}_0|$.

We cite the comments of a kind referee about paper (6):
I expect this paper to have an impact on the field comparable to the monumental work of Libschaber and Maurer in the early 80's on the period-doubling route to chaos.

VIII. EFFECT OF IMPERFECT VERTICAL ALIGNMENT ON TURBULENT RBC

We made a quantitative study of the effect of a slight tilt angle β relative to gravity of the axis of a cylindrical sample. This work is being published in paper (7). The measurements were made for R up to 10^{11} with our medium and large samples, both with $\Gamma \simeq 1$. In contrast to the experiences reported by [9] for a similar sample but with $\Gamma \simeq 0.5$, we found no long relaxation times.

For $R = 9.4 \times 10^{10}$ we measured the Nusselt number $\mathcal{N}$ as a function of β and obtained a small β dependence given by $\mathcal{N}(\beta) = \mathcal{N}_0[1 - (3.1 \pm 0.1) \times 10^{-2}|\beta|]$ when β is in radian. This depression of $\mathcal{N}$ is about a factor of 50 smaller than the result found by [9] for their $\Gamma \simeq 0.5$ sample.

We measured side-wall temperatures at eight equally spaced azimuthal locations on the horizontal mid-plane of the sample and used them to obtain Reynolds numbers R_e^{cc} of the LSC. For the large sample and $R = 9.4 \times 10^{10}$ we found $R_e^{cc}(\beta) = R_e^{cc}(0) \times [1 + (1.85 \pm 0.21)|\beta| - (5.9 \pm 1.7)\beta^2]$. These results are consistent with measurements of the amplitude δ of the azimuthal side-wall temperature-variation at the mid-plane that gave $\delta(\beta) = \delta(0) \times [1 + (1.84 \pm 0.45)|\beta| - (3.1 \pm 3.9)\beta^2]$ for the same R. An important conclusion is that the increase of the speed (i.e. of R_e) with β of the LSC does not significantly influence the heat transport. Thus the heat transport must be determined primarily by the instability mechanism operative in the boundary layers, rather than by the rate at which "plumes" are carried away by the LSC. This mechanism apparently is independent of β.

Over the range $10^9 \lesssim R \lesssim 10^{11}$ the enhancement of R_e^{cc} at constant β due to the tilt could be described by a power law of R with an exponent of $-1/6$, consistent with a simple model that balances the additional buoyancy due to the tilt angle by the shear stress across the boundary layers.

Even a small tilt angle dramatically suppressed the azimuthal meandering and the sudden reorientations characteristic of the LSC in a sample with $\beta = 0$.

IX. STUDENTS AND POST-DOOCTORAL SCHOLARS SUPPORTED BY THIS GRANT

1.) Alexey Nikolaenko, undergraduate student. Now a graduate student at the University of California at Irvine.

2.) Eric Brown, graduate student, Ph.D 2006. Now a Kadanoff-Rice Postdoctoral Scholar at the James Franck Institute, University of Chicago.

3.) Denis Funfschilling, post-doctoral scholar. Now a staff member at CNRS, Nancy, France.

X. PUBLICATIONS SUPPORTED BY THIS GRANT

1.) *Nusselt number measurements for turbulent Rayleigh-Bénard convection*, N. Nikolaenko and G. Ahlers, Phys. Rev. Lett. **91**, 084501 (2003).

2.) *Plume motion and large-scale circulation in a cylindrical Rayleigh-Bénard cell*, D. Funfschilling and G. Ahlers, Phys. Rev. Lett. **92**, 194502 (2004).

3.) *Heat transport by turbulent Rayleigh-Bénard Convection in cylindrical cells with aspect ratio one and less*, A. Nikolaenko, E. Brown, D. Funfschilling, and G. Ahlers, J. Fluid Mech. **523**, 251 (2005).

4.) *Heat transport in turbulent Rayleigh-Benard convection: Effect of finite top- and bottom-plate conductivity*, E. Brown, A. Nikolaenko, D. Funfschilling, and G. Ahlers, Phys. Fluids **17**, 075108 (2005).

5.) *Heat transport by turbulent Rayleigh-Benard Convection in cylindrical cells with aspect ratio one and larger*, D. Funfschilling, E. Brown, A. Nikolaenko, and G. Ahlers, J. Fluid Mech. **536**, 145 (2005).

6.) *Reorientation of the large-scale circulation in turbulent Rayleigh-Bénard convection*, E. Brown, A. Nikolaenko, and G. Ahlers, Phys. Rev. Lett. **95**, 084503 (2005).

7.) *Search for slow transients, and the effect of imperfect vertical alignment, in turbulent Rayleigh-Bénard convection*, G. Ahlers, E. Brown, A. Nikolaenko, J. Fluid Mech. **557**, 347 (2006).

8.) *Non-Oberbeck-Boussinesq effects in strongly turbulent Rayleigh-Bénard convection*, G. Ahlers, E. Brown, F. Fontenele Araujo, D. Funfschilling, S. Grossmann, and D. Lohse, J. Fluid Mech. **569**, 409 (2006).

9.) *Rotations and cessations of the large-scale circulation in turbulent Rayleigh-Bénard convection*, E. Brown and G. Ahlers, J. Fluid Mech. **568**, 351 (2006).

10.) *Effect of the Earth's Coriolis force on the large-scale circulation of turbulent Rayleigh-Bénard convection*, E. Brown and G. Ahlers, Phys. Fluids **18**, 125108 (2006).

XI. INVITED TALKS ON RESEARCH SUPPORTED BY THIS GRANT

1.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Leiden, Netherlands, High Rayleigh number turbulent convection workshop, 2003.

2.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, University of Twente, Netherlands, Oct. 2004.

3.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Center for Interdisciplinary Research in Fluid Dynamics, Santa Barbara, CA, May 13 2004.

4.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Confernce on Multi-scale Interactions in Turbulent Flow, Santa Fe, NM, July 18 2005.

5.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, PMMH, Paris, France, Sept. 18 2005.

6.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Colloquium, Dept. of Physics, Univ. of Calif. at Santa Barbara, Oct. 11 2005.

7.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Colloquium, James Frank Institute, U. Chicago, April 18 2006.

8.) *Recent experiments on turbulent Rayleigh-Benard convection by the Santa Barbara group*, Guenter Ahlers, Invited Talk, Workshop on turbulent Rayleigh-Bénard convection, Trieste, Italy, Sept. 6 2006.

9.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Colloquium, Max Planck Institute, Goettingen, Germany, , Trieste, Italy, Sept. 12 2006.

10.) *Turbulent Rayleigh-Benard convection*, Guenter Ahlers, Distinguished Lecture, Dept. of Mathematics, Arizona State University, Oct. 19 2006.

XII. CONTRIBUTED TALKS SUPPORTED BY THIS GRANT

We always contribute talks at the March APS meeting and at the annual meeting of the Division of Fluid Dynamics of the APS. At each of these, typically one or two are on work supported by the Department of Energy. We do not list these talks explicitly.

XIII. REFERENCES

[1] Grossmann, S. & Lohse, D. 2004 Fluctuations in turbulent Rayleigh-Bénard convection: the role of plumes *Phys. Fluids* **16**, 4462, and references therein.
[2] Verzicco, R. 2004 Effects of non-perfect thermal sources in turbulent thermal convection. *Phys. Fluids* **16**, 1965–1979.
[3] Kraichnan, R. 1962 Turbulent thermal convection at arbitrary Prandtl number. *Phys. Fluids* **5**, 1374–1389.
[4] Chavanne, X., Chillà, B., Chabaud, B., Castaing, B., & Hebral, B. 2001 Turbulent Rayleigh-Bénard convection in gaseous and liquid He *Phys. Fluids* **13**, 1300–1320.
[5] Ahlers, G. 2000 Effect of Sidewall Conductance on Heat-Transport Measurements for Turbulent Rayleigh-Benard Convection. *Phys. Rev. E* **63**, 015303-1–4(R).
[6] X. -L. Qiu, and P. Tong, Phys. Rev. Lett. **87** 094501 (2001).
[7] S. Cioni, S. Ciliberto, and J. Sommeria, J. Fluid Mech. **335**, 111 (1997).
[8] Niemela, J. & Sreenivasan, K. R. 2003 Confined turbulent convection. *J. Fluid Mech.* **481**, 355–384.
[9] Chillà, F., Rastello, M., Chaumat, S., & Castaing, B. 2004b Long relaxation times and tilt sensitivity in Rayleigh-Bénard turbulence. *Euro. Phys. J. B* **40**, 223–227.